Modal Pushover Analysis of SAC Buildings

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Abstract

Evaluated is the accuracy of the modal pushover analysis in estimating the seismic demands for six SAC buildings. These results are compared with those obtained by nonlinear response history analysis and three force distributions in FEMA-273.

Introduction

The nonlinear static procedure (NSP) or pushover analysis in FEMA-273 [FEMA, 1997] has become a standard procedure in current structural engineering practice. Seismic demands are computed by nonlinear static analysis of the structure, which is subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a target displacement is reached. None of the current invariant force distributions can account for the contribution of higher modes—higher than the fundamental mode—to the response or for redistribution of inertial forces because of structural yielding. To overcome these limitations several researchers have proposed adaptive force distributions that follow more closely the time-variant distributions of inertia forces (Fajfar and Fischinger, 1988; Bracci et al., 1997; Gupta and Kunnath, 2000). Others have tried to address this issue by considering more than the fundamental vibration mode in standard pushover analysis (Paret et al., 1996; Sasaki et al, 1998, Gupta and Kunnath, 2000; Kunnath and Gupta, 2000; Matsumori et al., 2000).

Recently, a modal pushover analysis (MPA) procedure has been developed that includes the contributions of several modes of vibration (Chopra and Goel, 2001). This paper demonstrates the accuracy of the MPA procedure in estimating the seismic demands for SAC buildings and compares these results with those obtained for the same buildings by pushover analysis using three force distributions in FEMA-273.

Modal Pushover Analysis Procedure: Summary

Summarized below are a series of steps used to estimate the peak inelastic response of a symmetric-plan, multistory building about two orthogonal axes to earthquake ground motion along an axis of symmetry using the MPA procedure developed by Chopra and Goel (2001):

1. Compute the natural frequencies, ω_n and modes, $\boldsymbol{\phi}_n$, for linearly elastic vibration of the building (Fig. 1).

2. For the nth-mode, develop the base shear-roof displacement, $V_{bn} - u_{rn}$, pushover curve for force distribution

   $$\mathbf{s}_n^* = \mathbf{m}\boldsymbol{\phi}_n$$

   where $\mathbf{m}$ is the mass matrix of the structure. These force distributions for the first three modes are shown schematically in Fig. 2 and the pushover curves for the first two modes in Fig. 3. For the first mode, gravity loads, including those present on the interior (gravity) frames, were applied prior to the pushover analysis. The resulting P-delta effects lead to negative post-yielding stiffness of the pushover curve (Fig. 3a). The gravity loads were not included in the higher mode pushover curves, which generally do not exhibit negative post-yielding stiffness (Fig. 3b).

3. Idealize the pushover curve as a bilinear curve (Fig. 4). If the pushover curve exhibits negative post-yielding stiffness, idealize the pushover curve as elastic-perfectly-plastic.

4. Convert the idealized pushover curve to the force-displacement, $F_{sn}/L_n - D_n$, relation (Fig. 4b) for the *n*th -"mode" inelastic SDF system by utilizing

$$\frac{F_{sny}}{L_n} = \frac{V_{bny}}{M_n^*} \qquad D_{ny} = \frac{u_{rny}}{\Gamma_n \phi_{rn}}$$

in which M_n^* is the effective modal mass, ϕ_{rn} is the value of $\boldsymbol{\phi}_n$ at the roof, and $\Gamma_n = \boldsymbol{\phi}_n^T \mathbf{m}\mathbf{1} \big/ \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\phi}_n$.

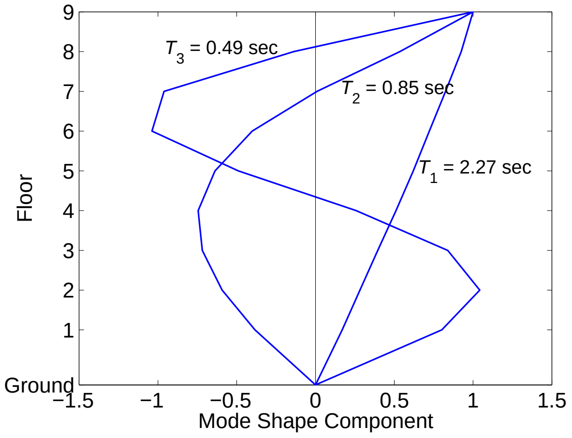


Fig. 1: First three natural-vibration periods and modes of the 9-story SAC-Los Angeles Building

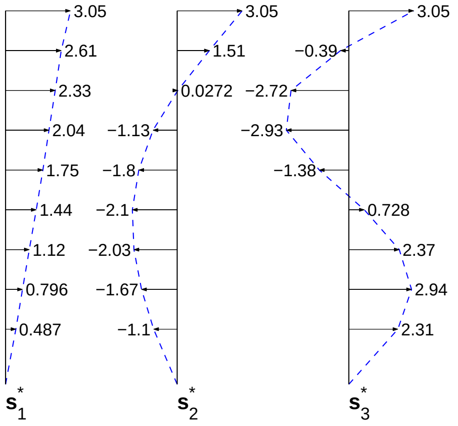


Fig. 2: Force distributions $\mathbf{s}_n^* = \boldsymbol{m}\boldsymbol{\phi}_n$, n = 1, 2, and 3 for the 9-story SAC-Los Angeles Building

5. Compute peak deformation D_n of the nth-"mode" inelastic SDF system defined by the force-deformation relation of Fig. 4b and damping ratio ζ_n. The elastic vibration period of the system is

$$T_n = 2\pi \left(\frac{L_n D_{ny}}{F_{sny}} \right)^{1/2}$$

For an SDF system with known T_n and ζ_n, D_n can be computed by nonlinear response history analysis (RHA) or from the inelastic design spectrum (Chopra, 2001, Section 7.11).

6. Calculate peak roof displacement u_{rn} associated with the *n*th-"mode" inelastic SDF system from

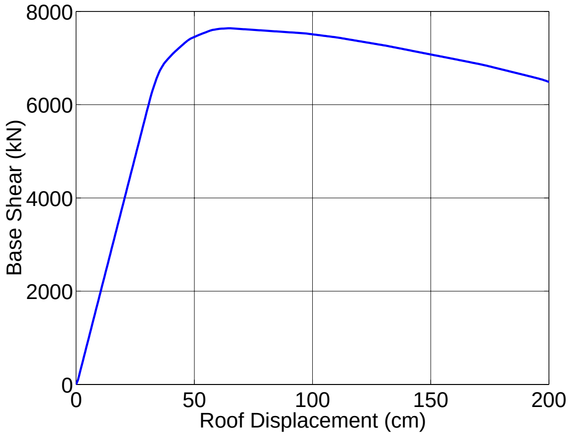
$$u_{rn} = \Gamma_n \phi_{rn} D_n$$

7. From the pushover database (Step 2), extract values of desired responses r_n: floor displacements, story drifts, plastic hinge rotations, etc.

8. Repeat Steps 3-7 for as many modes as required for sufficient accuracy. Typically, the first two or three "modes" will suffice.

9. Determine the total response (demand) by combining the peak "modal" responses using the SRSS rule:

$$r \approx \left(\sum_{n} r_n^2 \right)^{1/2}$$


(a) "Mode" 1 Pushover Curve

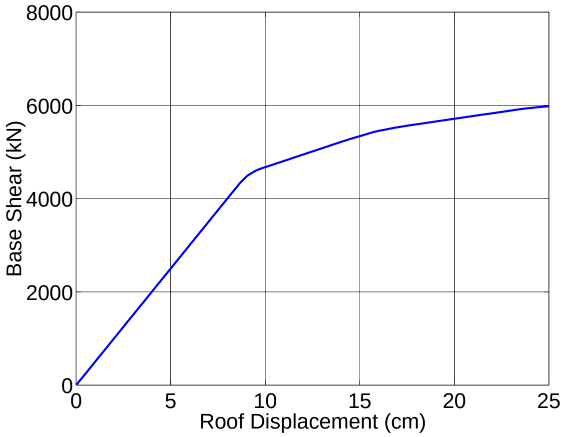
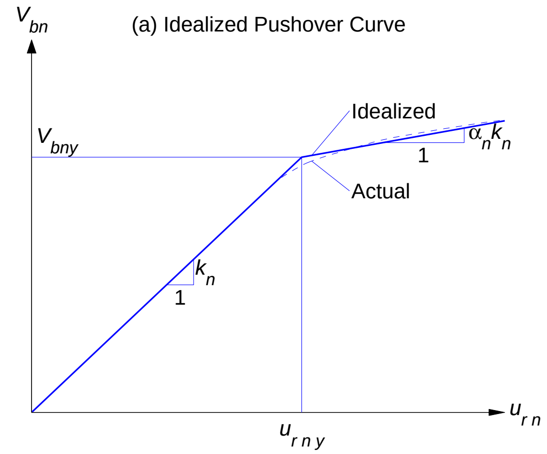

(b) "Mode" 2 Pushover Curve

Fig. 3: "Modal" pushover curves for the 9-story SAC-Los Angeles Building


(a) Idealized Pushover Curve

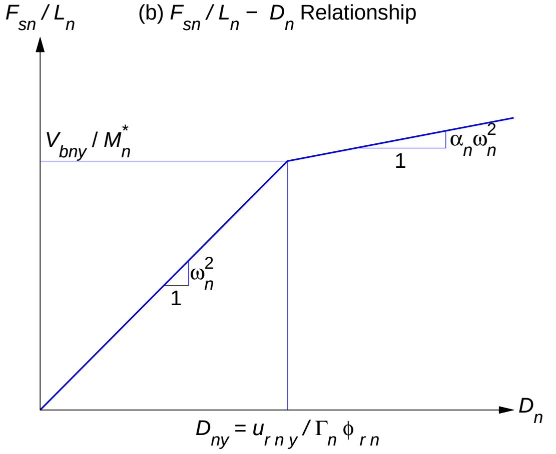

(b) $F_{sn}/L_n - D_n$ Relationship

Fig. 4: Properties of the *n*th-"mode" inelastic SDF system from the pushover curve

SAC Buildings and Ground Motions

SAC commissioned three consulting firms to design 3-, 9-, and 20-story model buildings according to the local code requirements of three cities: Los Angeles (UBC 1994), Seattle (UBC, 1994), and Boston (BOCA, 1993). Described in detail in Gupta and Krawinkler (1999), the structural systems of these model buildings consisted of perimeter steel moment-resisting frames (SMRF). The N-S perimeter frames of 9- and 20-story buildings are analyzed in this paper.

For all three locations, sets of 20 ground motion records were assembled representing probabilities of exceedance of 2% and 10% in 50 years (return periods of 2475 and 475 years, respectively) (Somerville et al., 1997). The 2/50 set of records are used in the subsequent analysis.

Story Drift Demands-Nonlinear Response History Analysis

The dynamic performance characteristics of SAC buildings were evaluated in a comprehensive study of story drift demand predictions through nonlinear RHA (Gupta and Krawinkler, 1999). Studied were the distribution of story drift demands over the height of the structures, the relation between story drift and roof drift, and the effect of modeling accuracy. Shown in Fig. 5 are the median values of story drift demands over the height of the buildings for Pre-Northridge M1 models of the structures. "Median" refers to the exponent of the mean of the natural log of the demand values due to 20 ground motions.

The distribution of story drift demands over the height of the structure (Fig. 5) is as follows:

- is strongly dependent on the structural characteristics;
- increases in upper stories of Boston (BO) structures, especially in the 20-story building because higher modes dominate the response;
- is more-or-less uniform in lower half of Seattle (SE) structures and concentrated in the upper five stories; and
- is more-or-less uniform in the upper half of Los Angeles (LA) structures, however, increases in the lower part of the buildings with the strongest concentration in the lowest 6 stories in 20-story buildings.

The story drifts presented cover a wide range of response from slightly beyond yielding—in the case of Boston structures—to very large demands—in the case of Los Angeles buildings—that represent response far into the inelastic range.

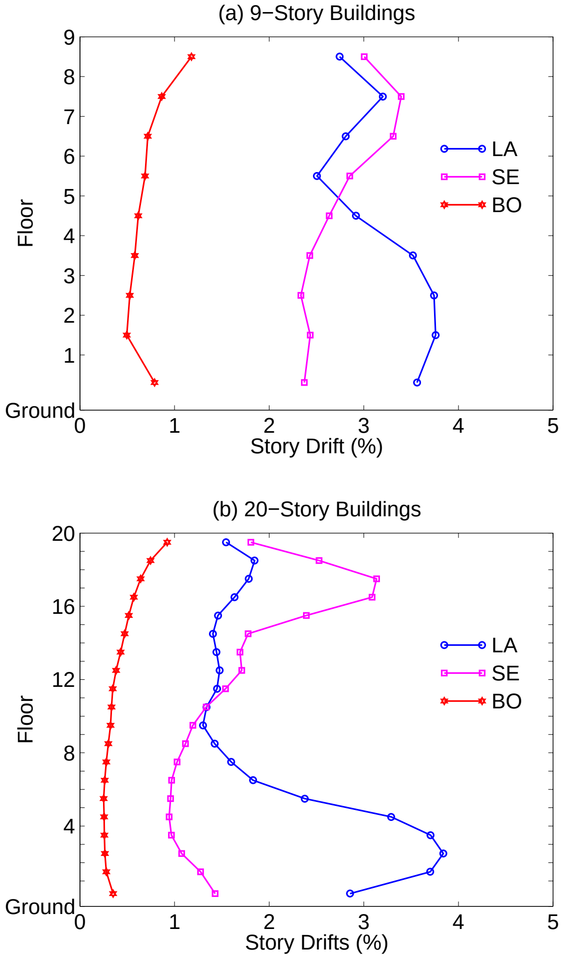


Fig. 5: Median story drift demands determined by nonlinear RHA: (a) 9-story buildings; (b) 20-story buildings (adapted from Gupta and Krawinkler, 1999)

Comparison of MPA and Nonlinear RHA Results

The MPA procedure was implemented for each of the six buildings and for each of the 20 ground motions. Contributions of the first three "modes" or the first five "modes" were considered for the 9-story buildings and 20-story buildings, respectively. The combined values of story drifts were computed for the 9-story building including one, two, or three "modes" and for the 20-story building including one, three, or five "modes." Figure 6 shows these median values of story drift demands together with the results of nonlinear RHA obtained from Fig. 5.

Figure 6 shows that the first "mode" alone is inadequate in estimating story drifts. However, by including the response contributions due to the second "mode" for the 9-story buildings and second and third "modes" for 20-story buildings the numbers are more accurate. With sufficient number of "modes" included, the height-wise distribution of story drifts estimated by MPA is generally similar to the trends noted from nonlinear RHA observed in the preceding section.

Accuracy of MPA Procedure

Figure 7 shows the errors in the story drift demands estimated by the MPA procedure, including contributions of sufficient number of modes: three modes for 9-story buildings and five modes for 20-story buildings. These results permit the following observations:

- The MPA procedure underestimates seismic demands in most stories of the Boston structures by about 20%; in few upper stories, the error may approach 30%.

- The MPA procedure estimates to acceptable accuracy seismic demands in the lower stories of the 9-story and 20 Seattle buildings, but underestimates demand near the top of the building by up to 30%.

- The MPA procedure is least accurate in estimating seismic demands for the LA buildings.

Errors in the MPA procedure for inelastic systems arise from several assumptions and approximations, but principally from: (1) neglecting coupling among modal coordinates associated with the modes of the corresponding linear system arising from yielding of the system; and (2) estimating the total response by combining the peak "modal" responses using the SRSS rule. The modal coordinates are indeed uncoupled for elastic buildings, and the MPA procedure is equivalent to standard response spectrum analysis (RSA). The RSA procedure, implemented in most commercial software, has become a standard analytical tool for the structural engineering profession. The principal source of approximation in this procedure is in using modal combination rules to combine the peak modal responses to estimate the total response. As these errors are considered acceptable by the profession, we compare next the errors in the MPA procedure with those in the RSA procedure.

For this purpose, elastic analysis of each building was implemented by RSA and RHA methods. The relative errors in story drift demands determined by the RSA procedure, also shown in Fig. 7, lead to the following observations.

- Depending on the structure and its location, RSA (with three modes for the 9-story buildings and five modes for 20-story buildings) underestimates the elastic response by 15% to 30%. The RSA errors are essentially uniform over the height of the structures;

- The errors in MPA are essentially the same as in RSA for the 9-story Boston structure because it remains essentially within the elastic range;

- The MPA errors are larger than RSA in the case of the 20-story Boston building because modest yielding occurs in upper stories;

- The MPA errors are larger than RSA in upper stories but smaller in lower stories in the case of Seattle buildings that undergo significant yielding; and

- The MPA errors vary irregularly over height and are much larger than RSA errors for the Los Angeles buildings because near-fault ground motions drive their response far into the inelastic range.

Comparison of Modal and FEMA Pushover Analyses

FEMA-273 Force Distributions

We consider only one step in the nonlinear static procedure in the FEMA-273 document (FEMA, 1997). The pushover curve, a plot of base shear versus roof displacement, is determined by nonlinear static analysis of the structure subjected to lateral forces with invariant distribution over height but gradually increasing values until a target value of roof displacement is reached. The gravity load is applied prior to the pushover analysis. The floor displacements, story drifts, joint rotations, plastic hinge rotations, etc., computed at the target displacement represent the seismic demands on the structure.

FEMA-273 specifies three distributions for lateral forces:

1. "Uniform" distribution: $s_j^* = m_j$, the mass at the *j*th floor level (where the floor number $j = 1, 2 \ldots N$);
2. Equivalent lateral force (ELF) distribution: $s_j^* = m_j h_j^k$ where h_j is the height of the *j*th floor above the base, and the exponent $k = 1$ for fundamental period $T_1 \le 0.5\ \mathrm{sec}$, $k = 2$ for $T_1 \ge 2.5\ \mathrm{sec}$; and varies linearly in between; and
3. SRSS distribution: $\mathbf{s}^*$ is defined by the lateral forces back-calculated from the story shears determined by response spectrum analysis of the structure, assumed to be linearly elastic.

Comparative Evaluation

Compared next are the story drift demands for each building determined by five analyses: pushover analysis using the three force distributions in FEMA-273, MPA considering three or five "modes," and nonlinear RHA. The target roof displacement in the analyses using FEMA force distributions was taken as equal to its value determined by the MPA procedure to achieve a meaningful comparison of the two methods, as shown in Fig. 8.

As clearly demonstrated in the figure, the height-wise variation of story drifts determined from the FEMA force distributions differs considerably from nonlinear RHA. Clearly, the FEMA force distribution procedure is inadequate; it does not predict:

- the increasing drifts in the upper stories of Boston structures;
- the concentration of large story drifts in the upper stories of Seattle structures (especially in the 20-story building); and
- the complex variation of story drifts over the height of the 20-story Los Angeles building.

Obviously, the MPA procedure performs much better than FEMA force distributions in estimating story drift demands.

Acknowledgments

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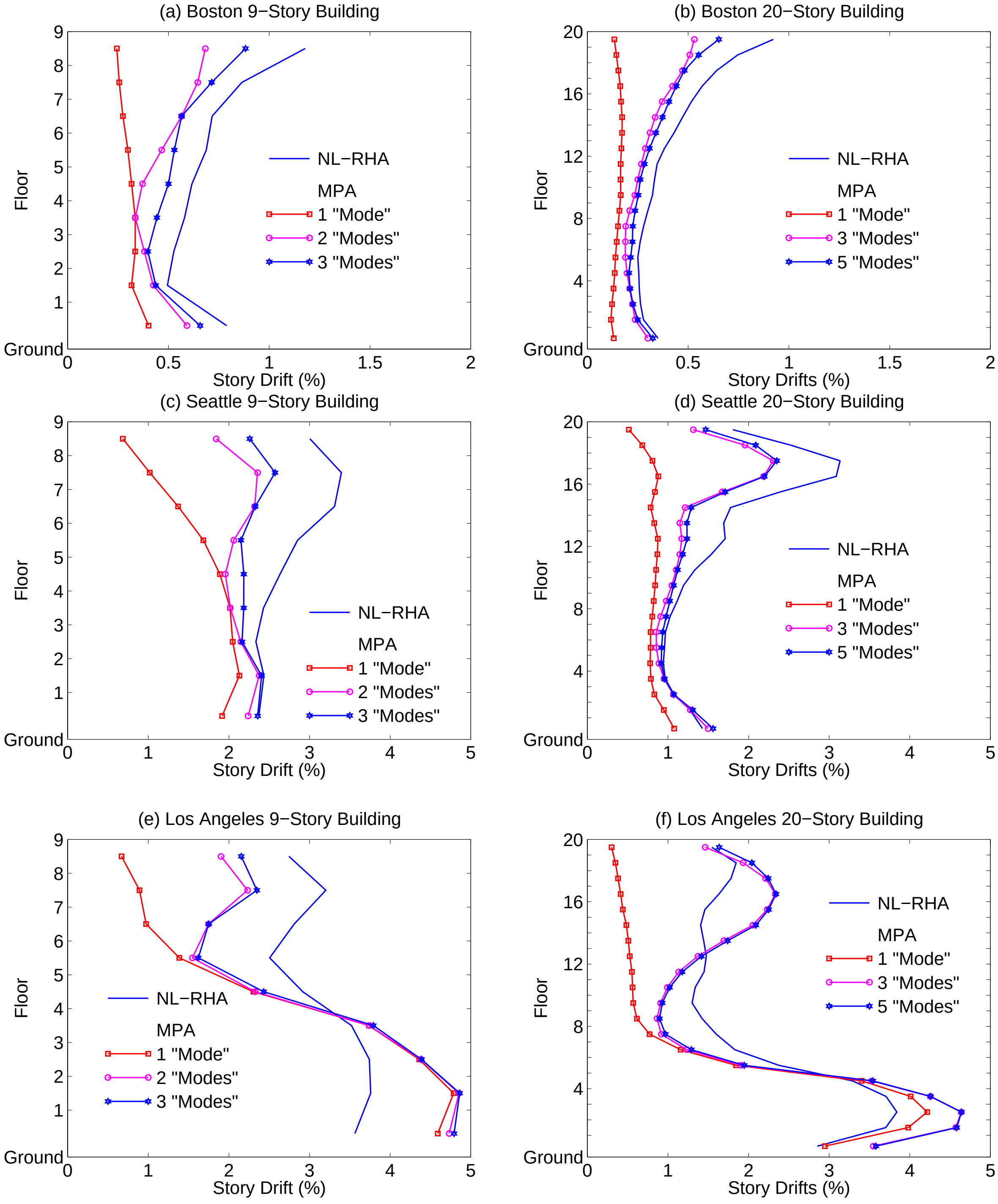
(a) Boston 9−Story Building

(b) Boston 20−Story Building

(c) Seattle 9−Story Building

(d) Seattle 20−Story Building

(e) Los Angeles 9−Story Building

(f) Los Angeles 20−Story Building

Fig. 6: Median story drift demands determined by MPA with variable number of "modes" and nonlinear RHA

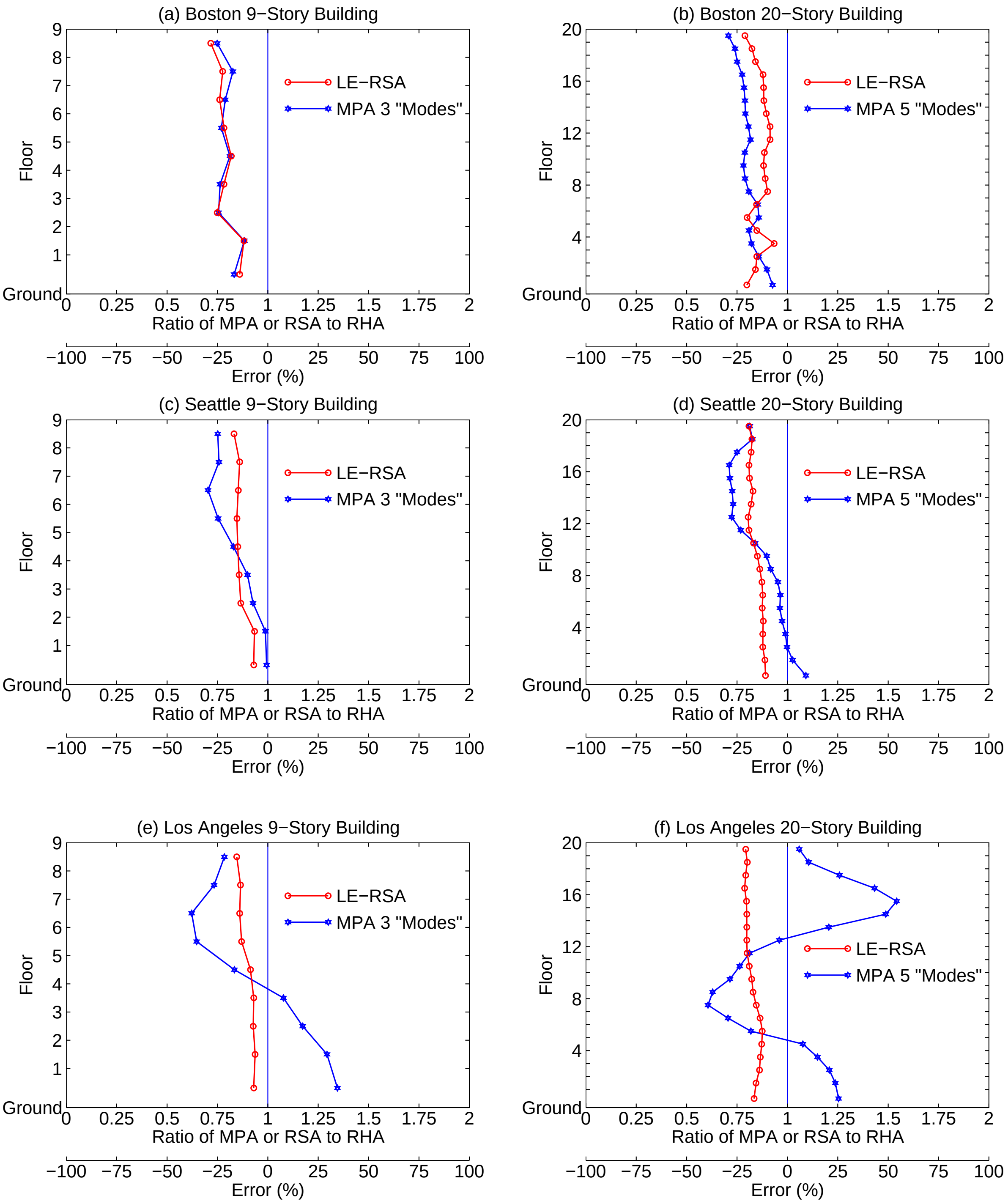

Fig. 7: Errors in the median story drift demands estimated by (1) MPA procedure for inelastic systems, and (2) RSA procedure for elastic systems

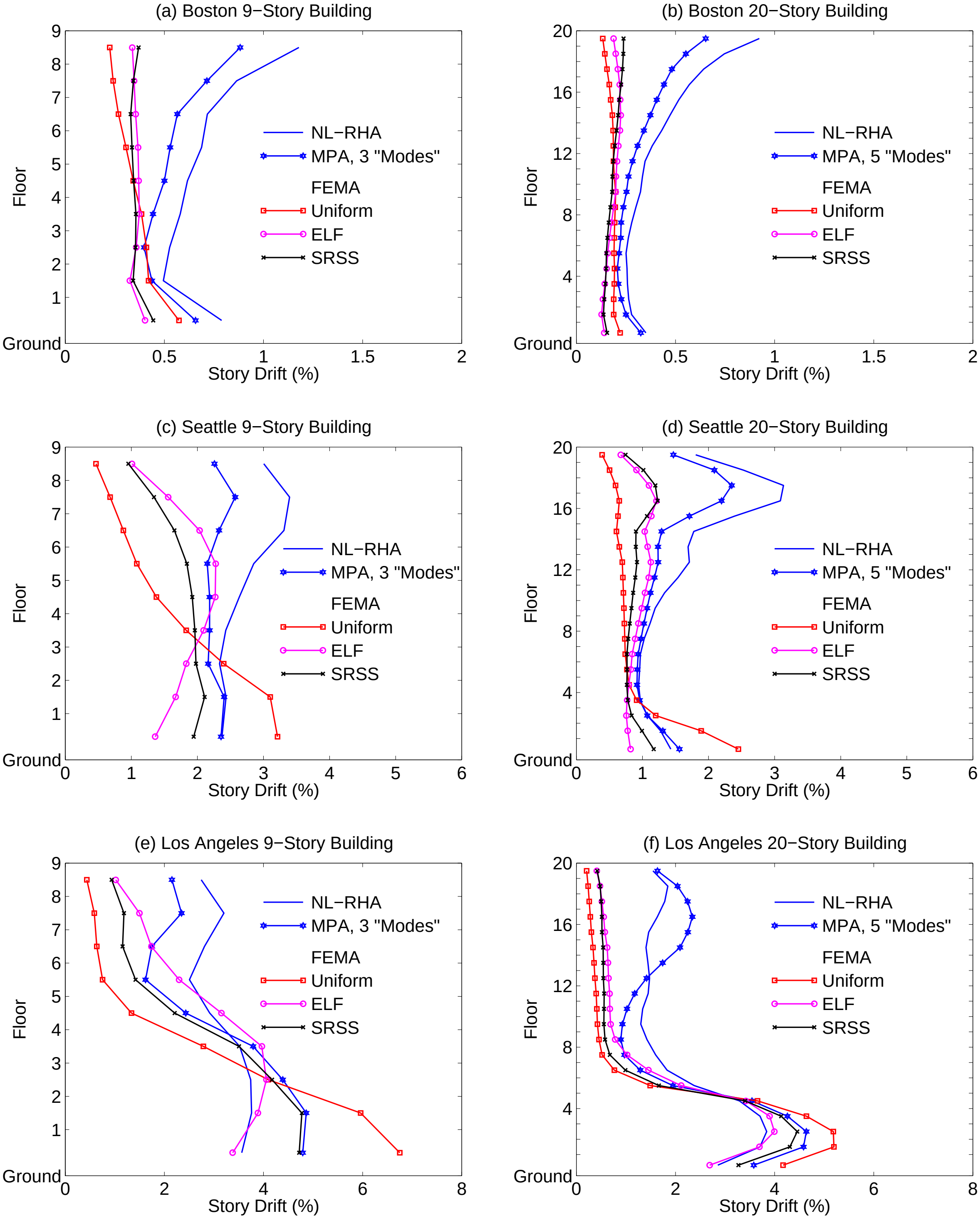

Fig. 8: Comparison of median story-drift demands determined by five procedures: pushover analysis using three force distributions in FEMA-273, MPA, and nonlinear RHA